

Bremsstrahlung in dark matter annihilation

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Abstract

We show that the energy spectra from dark matter (DM) annihilation into $f^+f^-\gamma$ (where f is a light fermion) via chirality preserving interactions are identical in 3 scenarios: (a) DM is a Majorana fermion and the particle exchanged is a scalar, (b) DM is a Majorana fermion and the particle exchanged is a vector and (c) DM is a scalar and the particle exchanged is a fermion. For cases (a) and (c), we also calculate the differential cross section to ℓ^+f^-V , where $V = W, Z, \gamma$, and ℓ is a light fermion that may or not be the same as f . The form of the cross section depends on whether the DM is a Majorana fermion or a scalar but in both cases its form is independent of V .

Intensive searches for dark matter (DM) via their annihilation signatures [1] have spurred interest in the effects of electromagnetic and electroweak bremsstrahlung on the rates for particular final states and on the resulting spectra [2, 3, 4].

In this Letter, we show that for 3 different interactions that are chirality preserving and appear in popular scenarios, dark matter annihilation to $f^+f^-\gamma$ in the static limit gives identical energy spectra. We then connect the common structure for the amplitude to an operator basis. Finally, for 2 of the 3 cases, we derive the differential cross section to ℓ^+f^-V , where $V = W, Z, \gamma$ and find its form to be different for Majorana DM and scalar DM, but independent of V .

Electromagnetic bremsstrahlung

We consider the annihilation final state $e^+e^-\gamma$ for the interaction terms in the table below. The exchanged particles S , W' and E (collectively denoted by X) are taken to be negatively charged and heavier than the corresponding DM particle $D = \chi, N$ and ϕ . We assume that D is a standard model singlet and its coupling to the electron to be chiral with the interaction involving e_R . For e_L , straightforward replacements can be made.

	Model 1	Model 2	Model 3
DM	Majorana χ	Majorana N	Scalar ϕ
Exchanged particle X	Scalar S	Vector W'	Fermion E
Interaction \mathcal{L}_I	$g'S^\dagger\bar{\chi}e_R$	$g'W_\mu'^\dagger\bar{N}\gamma^\mu e_R$	$g'\bar{E}\phi e_R$

The supersymmetric case of neutralino DM with selectron exchange is a good example of Model 1. The case of a new heavy charged W' gauge boson that couples e_R and a right-handed heavy Majorana neutrino N in variations of left-right theory falls into the class of Model 2 [5]. Scenarios of scalar DM which have been of interest recently are realizations of Model 3 [4].

For the annihilation process, $DD \rightarrow e(p_1)\bar{e}(p_2)\gamma(k, \epsilon)$, in the nonrelativistic limit we find a unified description of the energy spectra of the decay products for all three models. In standard

notation, the amplitude turns out to be

$$\mathcal{M} = -\frac{8eC\bar{u}_R(p_1)(\not{p}_2 \not{\epsilon} \not{k} \mp \not{k} \not{\epsilon} \not{p}_1)v_R(p_2)}{[(p_1 - p_2 + k)^2 - 4m_X^2][(p_1 - p_2 - k)^2 - 4m_X^2]}, \quad (1)$$

where the minus sign applies to Models 1 and 2 (with Majorana DM) and the plus sign applies to Model 3 (with scalar DM). Note that for the Majorana DM models only the initial state with vanishing total angular momentum participates in the annihilation. The overall constant

$$C = \begin{cases} \frac{i}{\sqrt{2}}g'^2 & \text{for Model 1,} \\ \frac{i}{\sqrt{2}}g'^2(2 + \frac{m_D^2}{m_X^2}) & \text{for Model 2,} \\ g'^2 & \text{for Model 3.} \end{cases}$$

Remarkably, we find the same structure for all 3 Models even though Model 2 involves the tri-gauge boson vertex $W'W'\gamma$. The amplitude obeys QED gauge invariance and its two terms do not interfere when photon polarizations are summed in the limit $m_e \rightarrow 0$. The spin-averaged annihilation rate is

$$v_{\text{rel}} \frac{d\sigma}{dx_1 dx_3} = \frac{1}{(2S_D + 1)^2} \frac{|eC|^2}{4\pi^3 m_D^2} \frac{[(1 - x_1)^2 + (1 - x_2)^2](1 - x_3)}{(1 - 2x_1 - r)^2(1 - 2x_2 - r)^2}, \quad (2)$$

where $r = m_X^2/m_D^2$. The scaling variables $x_i = E_i/m_D$ ($i = 1, 2$), $x_3 = E_\gamma/m_D$ are defined in the static center of mass frame so that $x_1 + x_2 + x_3 = 2$. The spin-averaged factor $\frac{1}{(2S_D+1)^2}$ is $\frac{1}{4}$ for Models 1, 2 and unity for Model 3. The photon energy distribution is obtained by integrating over $x_1 \in [1 - x_3, 1]$ [2]:

$$\begin{aligned} v_{\text{rel}} \frac{d\sigma}{dx_3} &= \frac{1}{(2S_D + 1)^2} \frac{|eC|^2}{32\pi^3 m_D^2} \frac{1 - x_3}{(1 + r - x_3)^2} \\ &\times \left(2x_3 \frac{x_3^2 + (1 + r - x_3)^2}{(1 + r)(1 + r - 2x_3)} - \frac{(1 + r)(1 + r - 2x_3)}{1 + r - x_3} \ln \frac{1 + r}{1 + r - 2x_3} \right). \end{aligned} \quad (3)$$

The unified formulas for the amplitude and the distribution for the 3 models are simple and interesting. We now study the structure of the amplitude. The numerator of the amplitude involves two pieces, each of which are QED gauge invariant. The first one is

$$\frac{1}{2}\bar{u}_R(p_1) \not{p}_2 \not{\epsilon} \not{k} v_R(p_2) = \bar{u}_R(p_1)(p_2 \cdot \epsilon \not{k} - p_2 \cdot k \not{\epsilon})v_R(p_2) \longleftarrow \mathcal{O},$$

where we have used the massless fermion on-shell condition and the last step identifies the structure with an operator [4],

$$\mathcal{O} = F^{\mu\nu} \bar{\psi}_R \gamma_\nu \partial_\mu \psi_R.$$

Similarly, the second piece is

$$\frac{1}{2}\bar{u}_R(p_1) \not{k} \not{\epsilon} \not{p}_1 v_R(p_2) = \bar{u}_R(p_1)(p_1 \cdot \epsilon \not{k} - p_1 \cdot k \not{\epsilon})v_R(p_2) \longleftarrow \mathcal{O}^\dagger,$$

where [4]

$$\mathcal{O}^\dagger = F^{\mu\nu}(\partial_\mu \bar{\psi}_R)\gamma_\nu \psi_R ,$$

is the conjugate of \mathcal{O} . On the other hand, we can use the Chisholm identity

$$\gamma^\alpha \gamma^\beta \gamma^\mu = g^{\alpha\beta} \gamma^\mu - g^{\alpha\mu} \gamma^\beta + g^{\beta\mu} \gamma^\alpha - i\epsilon^{\alpha\beta\mu\nu} \gamma_\nu \gamma_5 ,$$

to write

$$\begin{aligned} \bar{u}_R(p_1) \not{p}_2 \not{\epsilon} \not{k} v_R(p_2) &= \bar{u}_R(p_1)(p_2 \cdot \epsilon \not{k} - p_2 \cdot k \not{\epsilon} + \underbrace{k \cdot \epsilon \not{p}_2}_{\rightarrow 0} - i\epsilon^{p_2 \epsilon k \mu} \gamma_\mu \gamma_5) v_R(p_2) \\ &= -2i\epsilon^{p_2 \epsilon k \mu} \bar{u}_R(p_1) \gamma_\mu v_R(p_2) . \end{aligned}$$

Similarly,

$$\bar{u}_R(p_1) \not{k} \not{\epsilon} \not{p}_1 v_R(p_2) = +2i\epsilon^{p_1 \epsilon k \mu} \bar{u}_R(p_1) \gamma_\mu v_R(p_2) .$$

So we have the relations,

$$\mathcal{O} = F^{\mu\nu} \bar{\psi}_R \gamma_\nu \partial_\mu \psi_R = -i\tilde{F}^{\mu\nu} \bar{\psi}_R \gamma_\nu \partial_\mu \psi_R ,$$

$$\mathcal{O}^\dagger = F^{\mu\nu} (\partial_\mu \bar{\psi}_R) \gamma_\nu \psi_R = i\tilde{F}^{\mu\nu} (\partial_\mu \bar{\psi}_R) \gamma_\nu \psi_R ,$$

where $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$. Then,

$$\mathcal{O} + \mathcal{O}^\dagger = F^{\mu\nu} \partial_\mu (\bar{\psi}_R \gamma_\nu \psi_R) \equiv \mathcal{F} ,$$

$$i(\mathcal{O} - \mathcal{O}^\dagger) = \tilde{F}^{\mu\nu} \partial_\mu (\bar{\psi}_R \gamma_\nu \psi_R) \equiv \tilde{\mathcal{F}} .$$

Thus, either \mathcal{O} and \mathcal{O}^\dagger , or \mathcal{F} and $\tilde{\mathcal{F}}$ can be used as bases to describe the amplitude. Because the dual substitution interchanges $\mathbf{E} \leftrightarrow \mathbf{B}$, it also interchanges a right-polarized photon for a left-polarized photon. If the energy distribution sums up polarizations, both F and \tilde{F} give the same result. Note that since \mathcal{F} is CP even, it corresponds to Model 3 with scalar DM. On the contrary, the Majorana pair is CP odd, and the amplitude picks up $\tilde{\mathcal{F}}$.

Electroweak bremsstrahlung

For the annihilation of Majorana DM χ via scalar exchange, we derive a universal amplitude for vector boson V emission $\chi + \chi \rightarrow f(p_1) \bar{\ell}(p_2) V(k, \epsilon)$ in the static limit. The vector boson can be W, Z or γ . As $v_{\text{rel}} \rightarrow 0$, only the composite of $\chi\chi$ with zero total angular momentum gives a contribution, and we get an amplitude of the form of Eq. (1) with the minus sign selected, and C replaced by C_V (in which we collect the model-dependent couplings and coefficients). As usual, we assume that the chirality of the massless lepton field is conserved. Note that the two terms in the

numerator do not interfere only when $k^2 = 0$ (as for the photon $V = \gamma$). QED-like gauge invariance (i.e. $\mathcal{M} \cdot k = 0$) is satisfied even for $k^2 = m_V^2 \neq 0$. We calculate the annihilation cross section to be

$$v_{\text{rel}} \frac{d\sigma}{dx_1 dx_3} = \frac{|eC_V|^2}{16\pi^3 m_\chi^2} \frac{[(1-x_1-\delta_V)^2 + (1-x_2-\delta_V)^2 - 2\delta_V(1-x_3+\delta_V)](1-x_3+\delta_V)}{(1-2x_1-r)^2(1-2x_2-r)^2}, \quad (4)$$

where $r = m_X^2/m_\chi^2$ and $\delta_V = \frac{1}{4}m_V^2/m_\chi^2$. In the case of W/Z emission, we assume a common mass m_X for all $SU(2)$ partners of the heavy exchanged particles. Note that $x_3 = E_V/m_\chi$ and x_1 lie in the ranges $[2\sqrt{\delta_V}, 1+\delta_V]$ and $[0, 1-\delta_V]$, respectively. The energy distribution of V can be obtained by integrating over x_1 within the limits $x_1^\pm = (2-x_3 \pm \sqrt{x_3^2 - 4\delta_V})/2$, and the energy distribution of f can be obtained by integrating over x_3 from $1-x_1+\delta_V/(1-x_1)$ to $1+\delta_V$. On setting $\delta_V = 0$ we recover the cross section for $V = \gamma$ and C_γ is simply C given in the photon case. The values of C_W and C_Z are given by

$$C_W^2 = C_\gamma^2 \frac{T(T+1) - T_3 T_3'}{2 \sin^2 \theta_W}, \quad (5)$$

and

$$C_Z^2 = C_\gamma^2 \frac{(T_3 - Q \sin^2 \theta_W)^2}{\sin^2 \theta_W \cos^2 \theta_W}, \quad (6)$$

where T, T_3, T_3' are the weak-isospin numbers of the exchanged particles and θ_W is the weak mixing angle. Since the spinors in Eq. (1) can have either L or R chirality depending on the underlying physics, the weak-isospin numbers must be chosen accordingly. The exchanged particle S of Model 1 is an $SU(2)$ singlet with $Q = -1$.

For scalar DM ϕ that annihilates via fermion exchange, a relative sign is flipped in the numerator of the amplitude, and 2 additional terms appear for $V = W, Z$, but not for $V = \gamma$:

$$\mathcal{M} = -\frac{8eC_V \bar{u}_h(p_1)(\not{p}_2 \not{\epsilon} \not{k} + \not{k} \not{\epsilon} \not{p}_1 + \not{k} \not{\epsilon} \not{k} - k^2 \not{\epsilon})v_h(p_2)}{[(p_1 - p_2 + k)^2 - 4m_X^2][(p_1 - p_2 - k)^2 - 4m_X^2]}, \quad (7)$$

where $h = L, R$, and C_W and C_Z are as in Eqs. (5) and (6). The corresponding annihilation cross section is

$$v_{\text{rel}} \frac{d\sigma}{dx_1 dx_3} = \frac{|eC_V|^2}{4\pi^3 m_\phi^2} \frac{N}{(1-2x_1-r)^2(1-2x_2-r)^2}, \quad (8)$$

$$N = [(1-x_1-\delta_V)^2 + (1-x_2-\delta_V)^2 + 2\delta_V(1+x_3-2\delta_V)](1-x_3+\delta_V) \\ + 2\delta_V(1-x_1-\delta_V)(1-x_2-\delta_V),$$

with kinematic variables defined as before (and m_χ replaced by m_ϕ). For the interaction Lagrangian of Model 3 in the table, the exchanged particle E is an $SU(2)$ singlet with $Q = -1$. The realization of Model 3 in Ref. [4] uses an $SU(2)$ -doublet (N^0, E^-) for the exchanged particles with $T = \frac{1}{2}$, $T_3 = -\frac{1}{2}$, $T_3' = \frac{1}{2}$ and $Q = -1$.

The difference in the W/Z spectra for Models 1 and 3 arises from terms proportional to δ_V . In both models, the energy distributions for the photon and the W/Z differ substantially due to

kinematic effects if D is just above the W/Z thresholds, while for a high mass D the difference is suppressed by δ_V .

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